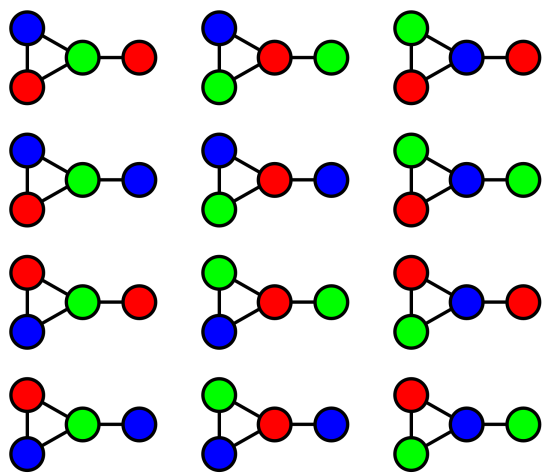
Tensor Networks in the wild

We have so far focused on using TN to describe many-body quantum states. But the toolbox is applicable far and wide. Here I will describe 2 different problems where a TN perspective is useful.

- Graph coloring



Given a graph G and an integer k, how many different colorings are allowed? [no 2 vertices have the same color]

$\Rightarrow$ Chromatic polynomial.

$$P(G,k) = k^n + a_2 k^{n-1} + a_3 k^{n-2} + \cdots$$

$(n = |G|)$

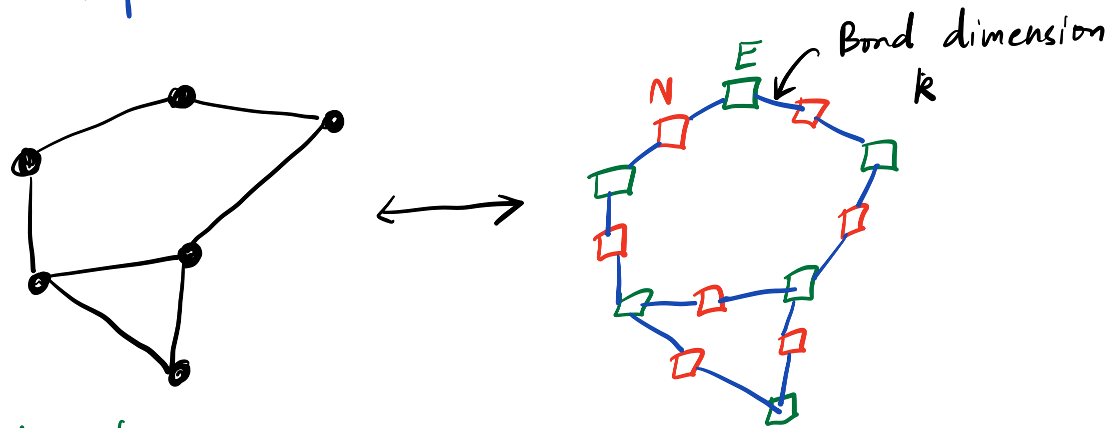
The answer is very hard to compute for general graph.

- Is a 4-regular planar graph 3-colorable ?
  - NP-hard !

- What is the number of q-coloring for a planar graph for $q \geq 3$? $\to$ #P hard.

Related to TN contraction !

In fact for any graph G, $P(G,k)$ can be represented as the answer of a TN contraction ?



on the vertex

$$\boxed{E} = \begin{cases} 1 & \text{if all the indices are equal} \\ 0 & \text{otherwise.} \end{cases}$$

along the edge.

$$\boxed{N} = \begin{cases} 1 & \text{if the 2 indices don't match} \\ 0 & \text{otherwise.} \end{cases}$$

always a rank-2 tensor

$\Rightarrow$ TN contraction for generic planar graphs is very very hard !

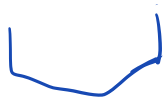
• TN diagrams for quantum processes

example Quantum teleportation :

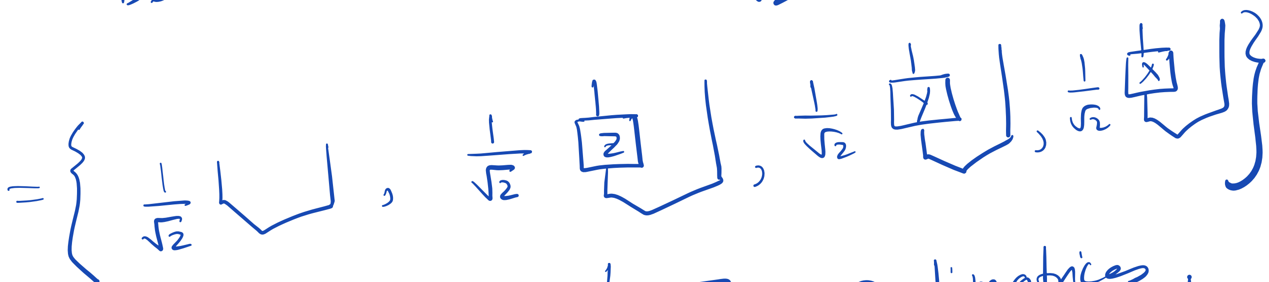
1. Alice and Bob share a Bell pair . AB.

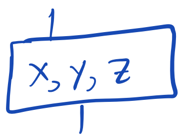
$$|\Omega\rangle_{AB} = \frac{|00\rangle + |11\rangle}{\sqrt{2}} \xrightarrow{\text{matrix}} \frac{\mathbb{1}}{\sqrt{2}}$$

$\equiv \frac{1}{\sqrt{2}}$

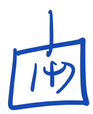


Bell Basis states : $\frac{|00\rangle \pm |11\rangle}{\sqrt{2}}$; $\frac{|01\rangle \pm |10\rangle}{\sqrt{2}}$



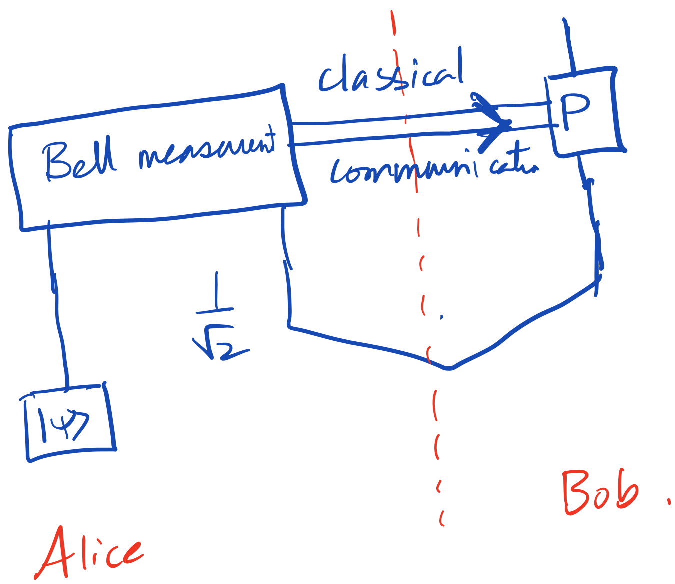


→ Pauli matrices .

2. Alice has a quantum state Q: 

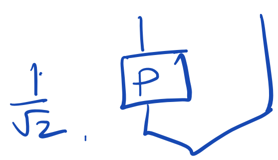
3. Alice measures Q and A in Bell basis and informs Bob of their measurement

4. Bob applies a Pauli correction based on Alice's information. Bob's qubit B is now in state $|\psi\rangle$!

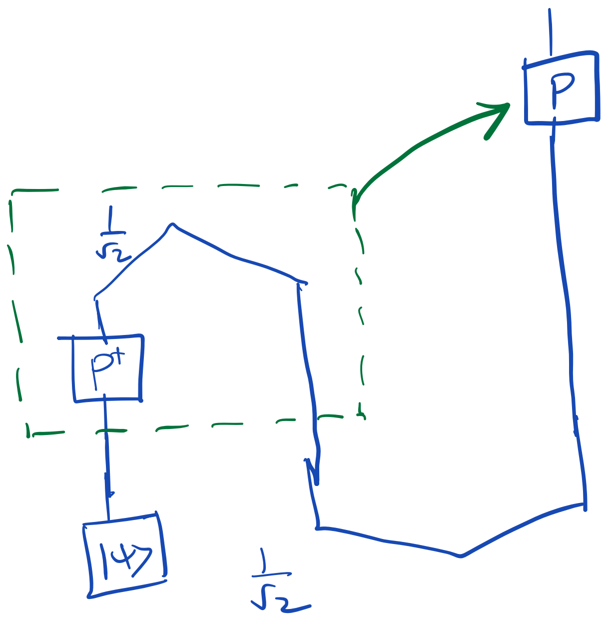


How do we know this correct?

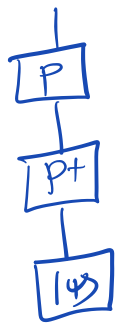
How can I model measurement: contraction with one of the eigenstates of my measurement operator:



!



$\equiv \quad \frac{1}{2}$



$= \frac{1}{2} |\psi\rangle$!

This trajectory happens with probability $\left(\frac{1}{2}\right)^2$ $= 1/4$, as we expect from teleportation.